

Scheduling and Cooperative Control of Electric Vehicles' Charging at Highway Service Stations

Azwirman Gusrialdi, Zhihua Qu and Marwan A. Simaan

Abstract—Due to their limited ranges, electric vehicles (EVs) need to be periodically charged during their long-distance travels on a highway. Compared to the fossil-fuel powered vehicles, the charging of a single EV takes much more time (up to 30 mins versus 2 mins). As the number of EVs on highways increases, adequate charging infrastructure needs to be put in place. Nonetheless waiting times for EVs to get charged at service stations could still vary significantly unless an appropriate scheduling coordination is in place and individual EVs make correct decisions about their choice of charging locations. This paper attempts to address both the system-level scheduling problem and the individual control problem, while requiring only distributed information about EVs and their charging at service stations along a highway. Specifically, we first develop a higher-level distributed scheduling algorithm to optimize the operation of the overall charging network. The scheduling algorithm uses only local information of traffic flows measured at the neighboring service stations (nodes), and it aims at adjusting the percentage of the EVs to be charged at individual stations so that all the charging resources along the highway are well (uniformly) utilized and the total waiting time is minimized. Then, a lower level cooperative control law is designed for individual EVs to decide whether or not it should charge its battery when approaching a specific service station by meeting the published scheduling level while taking into account its own battery constraint. Analytical designs are presented and their performance improvement is illustrated using simulation.

I. INTRODUCTION

The electric vehicle (EV) technology has attracted much interest in recent years. Compared to conventional fossil-fuel power vehicles, EVs offer many benefits, such as high energy efficiency, low greenhouse gas emissions, and energy independence [1], [2]. From the consumers' point of view, two main issues related to the adoption of EVs are the *long charging times* which may take from 30 mins to 8 hours [3], and the *limited driving range* which may be 60 to 200 miles. It is predicted that the number of EVs driven will be not less than 3.1 million by 2020 [3]. The combination of increasing number of EVs on the road and the long charging times may have a significant impact on the total acceptable waiting times for the customers to get their EVs charged.

Research effort on EVs' charging has largely focused on two main issues: (1) optimal placement of charging stations, e.g. [4]–[6] and (2) development of EVs' charging scheduling

algorithm, e.g. [7]–[14]. This paper falls into the latter. Specifically, we consider a scenario where a number of EVs are on a highway with service stations at selected entrances/exits where drivers can charge their batteries. Clearly, as the number of EVs increases, waiting times to get charged at service stations could increase significantly. This becomes more crucial if an appropriate scheduling algorithm and a process by which individual EVs make correct decisions about their charging locations do not exist. Some of these problems have already been considered in the literature. A stochastic balancing routing algorithm is proposed in [9] for balancing the demand across a network for charging stations and for reducing the potential for long queues at some charging stations. The authors in [10] propose a scheduling algorithm to minimize the lower bound of the waiting time for EV charging in a large-scale road network by assuming the presence of a central server which knows all information about the road system, the electricity charging stations, and the EVs. The authors in [12] consider an EV charging control in a shopping center parking lot. A control algorithm is proposed to balance the loss of life of the distribution transformer for the facility and to maximize the quality of charging service such as customer waiting time. EV charging problem in shopping centers is also considered in [15] where the authors analyze a lower bound of the probability that the battery of EVs becomes empty on the road and the mean of pure waiting time for the trip and show via simulations that there is a trade-off between the probability and waiting time. The work in [13] presents a charging-station selection algorithm with a short waiting time based on local information (the current position of the EV, the remaining battery status, and its distance to the charging station) or global information (queue lengths of the charging stations) on a highway. Finally, the authors in [14] present an algorithm for directing EVs to charging stations for balancing the load among charging stations in an area while minimizing the EVs charging time. It is assumed that vehicles can communicate with the grid and a mathematical model based on queuing theory is developed to handle requests for charging vehicles at public charging station.

In this paper we develop algorithms for *distributed* real-time scheduling and cooperative control for individual EVs from a dynamical system's perspective so that all the charging stations along the highway are well (uniformly) utilized and the total waiting time minimized. The scheduling algorithm uses local information of traffic flow from neighboring service stations, namely the previous and next service stations and aims at adjusting the percentage of the EVs

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that need to be recharged at each station. The cooperative control algorithm is designed for an individual EV by means of infrastructure-to-vehicle and vehicle-to-vehicle wireless communication to decide whether or not it should charge its battery when approaching a specific service station by meeting its schedule while taking into account its own battery constraint.

The remainder of the paper is organized as follows. In section II an average dynamic model of EV flow passing through entrances/exits is described and the problem of distributed scheduling for the overall transportation network and a cooperative control for individual drivers to make their decisions are presented. A consensus-based distributed scheduling algorithm and distributed policy of decision making for individual drivers are presented and analyzed in section III and IV respectively. An illustrative example is included in section V, and conclusions are drawn in section VI.

II. MODELING & PROBLEM STATEMENT

Consider a highway (e.g., Florida Turnpike) along which there are a number of entrances/exits. At selected entrances/exits, there are service stations where EV drivers can charge their batteries. Due to their much-shorter ranges, EVs need to be periodically charged during long-distance travels, and hence both system-level scheduling and individual control algorithms should be designed properly.

In the remainder of this section, we first describe an average dynamic model of EV flow passing through entrances/exits. Then, a queueing model is presented for the average number of EVs at a given service station. Finally, the decision variables are identified to formulate the problems of designing algorithms for distributed scheduling for the overall charging network and for cooperative control for individual drivers to make their decisions.

A. An Average Model of Vehicle Flow

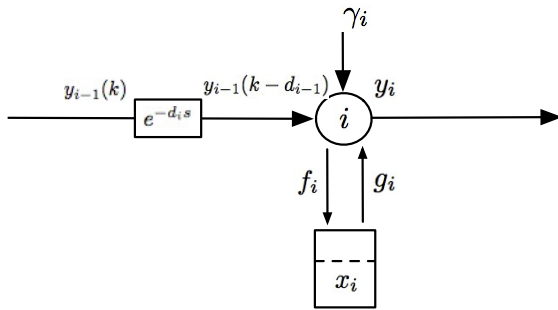


Fig. 1. A simple flow model of EVs at the i th entrance/exit

Consider the traffic flow in one direction and assume that the highway has a total of N nodes (entrances/exits). Then, a simple discrete time model of traffic flow is shown in figure 1, that is, the vehicle flow continuing further at the i th node and at time k is described by

$$y_i(k) = g_i(k) + [1 - p_i(k)] [y_{i-1}(k - d_{i-1}) + \gamma_i(k)], \quad (1)$$

where $y_i(k)$ is the EV flow continuing their travel from node i , d_{i-1} denotes the average travel time (in unit increments) from node $(i-1)$ to node i , γ_i is the net average flow entering node i from local roadways, $p_i(k)$ is the percentage of EVs that enters the service station at node i , and $g_i(k)$ is the EV flow out of the service station. If node i has no service station, $g_i(k) = 0$ and $p_i(k) = 0$.

B. A Queueing Model for Service Stations

Let $x_i(k) \geq 0$ denote the number of EVs at the service station of node i and at time k . Then, the vehicle flow on the highway interacts with the state of the service station according to

$$\begin{aligned} x_i(k+1) &= x_i(k) + f_i(k) - g_i(k), \\ f_i(k) &= p_i(k) [y_{i-1}(k - d_{i-1}) + \gamma_i(k)], \end{aligned} \quad (2)$$

where $f_i(k)$ is the average flow into the service station at node i , and $g_i(k)$ is the average flow out of the service station.

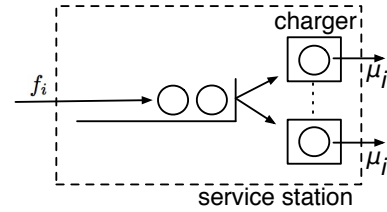


Fig. 2. Queueing at the service station of node i

As shown in figure 2, an EV arrives entering the service station will wait in a queue until a charging station becomes available for it to get charged. It is assumed that, at the service station of node i , there are c_i EV chargers (which always serve from the front of the queue) while the capacity for the EVs to wait in the queue is sufficiently large. The relationship among $f_i(k)$, $g_i(k)$ and $x_i(k)$ is modeled as an $M/M/c_i$ queue: the EV arrives at service station i with the mean rate of $f_i(k)$, state $x_i(k)$ evolves according to a Poisson process, and charging times have an exponential distribution with parameter μ_i (which is referred to as service rate). This $M/M/c_i$ queue can be graphically represented by the birth-death process illustrated by figure 3.

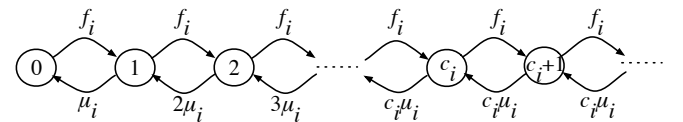


Fig. 3. A state diagram of a birth-death process ($M/M/c_i$ queue)

C. A Simple Energy Model of EVs

Electricity consumption of the v th EV is modeled as

$$e_{v,i+1}^- = e_{v,i}^+ - d_i r_v^-, \quad (3)$$

where $e_{v,i+1}^-$ denotes the battery status of the v th vehicle when it approaches node $(i+1)$, $e_{v,i}^+$ is its battery status

when it leaves node i , d_i is the average drive time from node i to node $(i + 1)$, and r_v^- is the battery consumption rate of the v -th vehicle. When the v th EV leaves node i , its battery status changes according to

$$e_{v,i}^+ = u_{v,i} e_{v,max} + (1 - u_{v,i}) e_{v,i}^-, \quad (4)$$

where $e_{v,max}$ is the capacity of the v th vehicle's battery, and $u_{v,i} \in \{0, 1\}$ is the decision variable given by

$$u_{v,i}(k) = \begin{cases} 1 & \text{if vehicle } v \text{ decides to charge} \\ & \text{at node } i \text{ and time } k \\ 0 & \text{otherwise.} \end{cases} \quad (5)$$

Model (4) implies that an EV will be fully charged once it decides to enter a service station.

D. Distributed Scheduling and Cooperative Control

The objective of this paper is twofold. First, we develop a higher-level distributed scheduling algorithm to optimize the operation of the overall transportation network. Second, a lower level cooperative control is designed for individual EV to decide whether it should charge its battery when approaching a specific service station by meeting the scheduling level while taking into account its own battery constraint as illustrated in figure 4.

The scheduling algorithm uses local information of traffic flow, specifically, the service station at node i and time k only requires the information of $y_{i-1}(k - d_{i-1})$, $x_{i-1}(k - 1)$, $\gamma_i(k)$, $x_i(k - 1)$ and $x_{i+1}(k - 1)$. It aims at adjusting $p_i(k)$ so that the average wait times at service stations reach a consensus, because such a consensus reflects the objective of optimizing charging services and highway operation so that all the charging stations along the highway are uniformly utilized and the total waiting time is minimized. Mathematically, the design can be represented as follows: find $p_i^*(k)$ such that

$$p_i(k) = p_i^*(k) \implies \frac{x_i(k)}{c_i \mu_i} \rightarrow \xi_0(k). \quad (6)$$

As the stability issue to be investigated in section III-C, the stability of the consensus value is ensured if the number of EVs at any of the service stations does not keep growing. At the steady state, this calls for the condition

$$\rho_i < c_i, \quad (7)$$

where $\rho_i(k) = f_i(k)/\mu_i$ is the so-called utilization of those chargers located at the service station of node i .

By the means of wireless communication, those EVs approaching node i at time k can form neighboring set $\mathcal{N}_i(k)$, and an EV from this set can interact with neighboring EVs and the next service station (i.e., acquiring the information of $p_i^*(k)$) so that it can negotiate (based on its own expected energy level) and determine which service station is best for it to get charged with minimum waiting time. Mathematically, this objective can be expressed as

$$\frac{\sum_v u_{v,i}(k)}{y_{i-1}(k - d_{i-1}) + \gamma_i(k)} \rightarrow p_i^*(k), \quad (8)$$

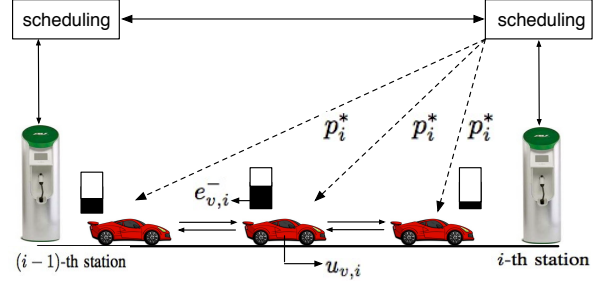


Fig. 4. Distributed scheduling and control of EVs' charging

where p_i^* is the current state of the consensus scheduling algorithm. The combination of the two distributed algorithms is (and will be shown) to improve performance of the overall system.

III. DISTRIBUTED SCHEDULING ALGORITHM

In this section, we use the steady state solution of $M/M/c_i$ queue to design a consensus law as the distributed scheduling algorithm for flow dynamics in equations (1) and (2). Convergence of consensus is shown, and stability of the consensus value is also analyzed.

A. Steady State Solution of $M/M/c_i$ Queue

As in [10], [16], we model the charging service at the i th service station as an $M/M/c_i$ queue. The model assumes that EVs arrive at service station i according to a Poisson process of mean arrival rate $f_i(k)$ and with an exponential distribution of service rate μ_i . Specifically, an $M/M/c_i$ queue is a stochastic process whose state space is the set $\mathbb{N} = \{0, 1, 2, 3, \dots\}$, that is, it is a Markov process of $\{x_i(k) = l : l \in \mathbb{N}\}$, where $x_i(k)$ is the state variable in equation (2). Let the steady-state solution be denoted as $\omega_i = [\omega_{i,0}, \omega_{i,1}, \dots]^T$, where

$$\omega_{i,l} = \lim_{k \rightarrow \infty} \text{Probability}\{x_i(k) = l\}.$$

The steady state solution satisfies constraint $\sum_{l=0}^{\infty} \omega_{i,l} = 1$ and equation $\omega_i Q_i = 0$, where Q_i is the transition matrix given by

$$\begin{bmatrix} -f_i & f_i & & & & & \\ \mu_i & -(f_i + \mu_i) & f_i & & & & \\ & & \ddots & & & & \\ & & & c_i \mu_i & -(f_i + c_i \mu_i) & f_i & \\ & & & & c_i \mu_i & -(f_i + c_i \mu_i) & f_i \\ & & & & & & \ddots \end{bmatrix}.$$

It is shown in [17] that the steady solution of $x_i(k)$ is given by

$$x_i = \frac{\rho_i^{c_i+1}}{c_i c_i!} \frac{1}{(1 - \rho_i/c_i)^2 \omega_{i,0} + \rho_i}, \quad (9)$$

$$\omega_{i,0} = \left[\sum_{n=0}^{c_i-1} \frac{\rho_i^n}{n!} + \frac{\rho_i^{c_i}}{c_i! (1 - \rho_i/c_i)} \right]^{-1},$$

where $\rho_i = f_i/\mu_i$ is the utilization of the chargers at service station i .

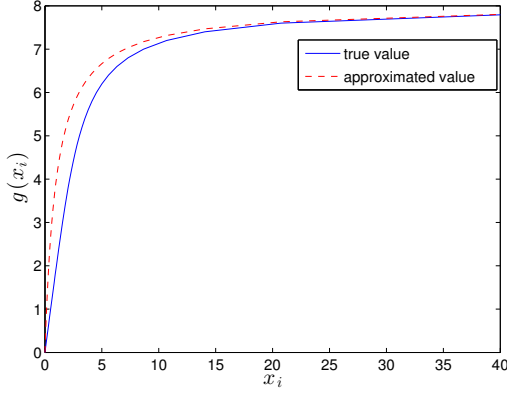


Fig. 5. Comparison between output function $g_i(x_i)$ and its approximation $\hat{g}_i(x_i)$: $c_i = 4$ and $\mu_i = 2$

It follows from (2) that, at the steady state, the output flow from the service station is given by $g_i = f_i$. Therefore, we have the following input-output relationship of equation (2) and its M/M/c queue:

$$x_i = \frac{g_i^{c_i+1}}{\mu_i^{c_i+1} c_i c_i! (1 - g_i/\mu_i c_i)^2 \omega_{i,0}} + \frac{g_i}{\mu_i},$$

$$\omega_{i,0} = \left[\sum_{n=0}^{c_i-1} \frac{g_i^n}{\mu_i^n n!} + \frac{g_i^{c_i}}{\mu_i^{c_i} c_i! [1 - g_i/(\mu_i c_i)]} \right]^{-1}, \quad (10)$$

which is identical to the result from applying the pointwise stationary approximation (PSA) method in [18], [19]. Should $c_i = 1$, the steady state solution reduces to

$$x_i = \frac{g_i}{\mu_i - g_i} \implies g_i = \mu_i \frac{x_i}{1 + x_i}. \quad (11)$$

Solving $g_i(x_i)$ from (10) is generally impossible except for the simple case of $c_i = 1$ which is shown in (11). Nonetheless, for the purpose of deriving an analytical input-output solution of $g_i(x_i)$ (so that analytical design can be proceeded in the next subsection), a single M/M/ c_i queue can be approximately decoupled into c_i M/M/1 queues and hence the input-output solution $g_i(x_i)$ to equation (10) can be approximated by

$$\hat{g}_i(x_i) = c_i \mu_i \frac{x_i}{1 + x_i}. \quad (12)$$

In figure 5, solution (12) is compared to the numerical solution to equation (10), and hence we know that solution (12) is appropriate for analytical design and analysis.

B. Distributed Estimation Network of Total Charging Needs

For the purpose of collecting local information about charging needs, it is assumed that, by the means of *vehicle-to-infrastructure* communication [20], an EV entering the highway at node i and at time $t = kT$ will register itself and hence be counted in $\gamma_i(k)$. Then, the following distributed observer is used to estimate the total charging needs: for

$t \in [kT, (k+1)T)$,

$$\begin{aligned} \dot{\xi}_1 &= \kappa \left(-\xi_1 + \left[\frac{2\xi_1}{3} + \frac{\xi_2}{3} \right] \right) \\ \dot{\xi}_i &= \kappa \left(-\xi_i + \left[\frac{\xi_{i-1}}{3} + \frac{\xi_i}{3} + \frac{\xi_{i+1}}{3} \right] \right), \\ \dot{\xi}_N &= \kappa \left(-\xi_N + \left[\frac{2\xi_{N-1}}{3} + \frac{\xi_N}{3} \right] \right) \end{aligned}, \quad (13)$$

where $i = 2, \dots, (N-1)$, κ is the controller gain,

$$\xi_i(kT) = N \beta_i \gamma_i(k)$$

and β_i is the average number of charging needed per vehicle for those vehicles passing through node i and computed based on information collected at node i such as their energy levels in (3) and their distances to go. Observer (13) can be expressed in the matrix form as

$$\dot{\xi} = \kappa(-I + D)\xi, \quad (14)$$

where $\xi = [\xi_1 \dots \xi_N]^T$, and matrix D given by

$$\begin{bmatrix} \frac{2}{3} & \frac{1}{3} & 0 & 0 & \dots & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \end{bmatrix}. \quad (15)$$

The following lemma shows convergence of observer (13).

Lemma 1: For a given $0 < \delta < 1$ and choosing the gain $\kappa \geq \frac{4}{\lambda_2 \delta T}$ where λ_2 is the second smallest eigenvalue of matrix $(I - D)$, distributed observer (13) converges to the following: for all $i \in \{1, \dots, N\}$

$$\xi_i(kT + \delta T) \rightarrow \sum_{j=1}^N \beta_j \gamma_j(k). \quad (16)$$

Proof: The exponential decay rate of the observer (14) is equal to $\kappa \lambda_2$. Furthermore, the settling time δT is given by $\delta T = 4(1/\kappa \lambda_2)$. Given that matrix D in (15) is row stochastic, column stochastic and irreducible, and by choosing the gain $\kappa \geq \frac{4}{\lambda_2 \delta T}$ for a pre-defined δ , it can then be ensured [21] that observer (14) asymptotically converges to a consensus as $\xi(kT + \delta T) \rightarrow a_o \mathbf{1}$, where $\mathbf{1} \in \mathbb{R}^n$ is the vector of 1s and consensus value a_o is given by $a_o = \mathbf{1}^T \xi(kT)/N$. ■

In the following subsection we present a distributed scheduling algorithm to determine $p_i(k)$ for two different cases depending on the value of $\xi_i((k-1)T + \delta T)$, which is the estimation of the total charging needs of the EVs at the previous time step, i.e. $\sum_{j=1}^N \gamma_j(k-1)$. Based on the estimated total charging needs of the network, the infrastructure determines the consensus value of the normalized queue at the service stations, i.e., $\xi_0(k)$ in (6) for the two cases.

C. Distributed Algorithm of Scheduling at Service Stations

First, we consider the case that $\xi_i((k-1)T + \delta T)$ satisfies the following condition

$$\xi_i((k-1)T + \delta T) < \sum_{j=1}^N c_j \mu_j. \quad (17)$$

By the means of *infrastructure-to-infrastructure* communication, the service stations adjust $p_i(k)$ according to

$$\begin{aligned} p_1^*(k) &= \frac{c_1\mu_1}{\alpha_1(k)} \left[\frac{\xi_0(k)}{3} - \frac{2z_1(k)}{3} + \frac{z_2(k)}{3} + \frac{c_1\mu_1 z_1(k)}{1 + c_1\mu_1 z_1(k)} \right] \\ p_i^*(k) &= \frac{c_i\mu_i}{\alpha_i(k)} \left[\frac{\xi_0(k)}{3} + \frac{z_{i-1}(k)}{3} - z_i(k) + \frac{z_{i+1}(k)}{3} + \frac{c_i\mu_i z_i(k)}{1 + c_i\mu_i z_i(k)} \right] \\ p_N^*(k) &= \frac{c_N\mu_N}{\alpha_N(k)} \left[\frac{\xi_0(k)}{3} - \frac{2z_N(k)}{3} + \frac{z_{N-1}(k)}{3} + \frac{c_N\mu_N z_N(k)}{1 + c_N\mu_N z_N(k)} \right] \end{aligned} \quad (18)$$

where $i = 2, \dots, (N-1)$, $\alpha_1(k) = \gamma_1(k)$, and $\alpha_j(k) = y_{j-1}(k - d_{j-1}) + \gamma_j(k)$ for $j = 2, \dots, N$. In addition, the value $\xi_0(k) \geq 0$ which acts as a leader is given by the solution of

$$\sum_{j=1}^N \frac{(c_j\mu_j)^2 \xi_0(k)}{1 + c_j\mu_j \xi_0(k)} = \xi_i((k-1)T + \delta T). \quad (19)$$

The following theorem shows that consensus (6) is guaranteed.

Theorem 1: Under distributed scheduling algorithm (18) and if condition (17) is satisfied, all the charging stations are asymptotically uniformly utilized, i.e., consensus (6) is achieved and the queues at service stations are bounded as $k \rightarrow \infty$.

Proof: Defining $z_i(k) \triangleq x_i(k)/c_i\mu_i$ and substituting $\hat{g}_i(x_i)$ in (12) into (2) yield

$$z_i(k+1) = z_i(k) + \frac{f_i(k)}{c_i\mu_i} - \frac{c_i\mu_i z_i(k)}{1 + c_i\mu_i z_i(k)}. \quad (20)$$

Substituting (18) into (20) yields the closed-loop dynamics:

$$z(k+1) = Dz(k) + \frac{1}{3}[\xi_0(k)\mathbf{1} - z(k)], \quad (21)$$

where $z = [z_1 \dots z_N]^T$, $\mathbf{1} \in \mathfrak{R}^n$ is the vector of 1s, and matrix D given by (15) is primitive and row stochastic. It is well known [21] that system (21) asymptotically converges to a consensus as $z(k) \rightarrow \xi_0(k)\mathbf{1}$.

Next, we determine $\xi_0(k)$ to ensure that all the incoming EVs will be charged, i.e., the following condition is satisfied:

$$\sum_{j=1}^N f_j(k) = \xi_i((k-1)T + \delta T) \quad (22)$$

where $f_i(k)$ is defined in (2). Substituting (18) into (2) and since $z_i(k) \rightarrow \xi_0(k)$, we then have

$$f_i(k) \rightarrow \frac{(c_i\mu_i)^2 \xi_0(k)}{1 + c_i\mu_i \xi_0(k)}.$$

Substituting the above value of $f_i(k)$ into (22) gives us the equation (19). Observe that

$$\lim_{\xi_0(k) \rightarrow \infty} \sum_{i=1}^N \frac{(c_i\mu_i)^2 \xi_0(k)}{1 + c_i\mu_i \xi_0(k)} = \sum_{i=1}^N c_i\mu_i.$$

The solution $\xi_0(k)$ given by (19) always exists under condition (17). Since $\lim_{\xi_0(k) \rightarrow \infty} f_i(k) = c_i\mu_i$, we have that $f_i(k) < c_i\mu_i$ and thus from (7) the queue at each service stations, i.e. $x_i(k)$, is bounded. ■

Note that from (18) and since $z_i(k) \rightarrow \xi_0(k)$, we have

$$p_i^* \rightarrow \frac{(c_i\mu_i)^2 \xi_0(k)}{\alpha_i(k) [1 + c_i\mu_i \xi_0(k)]}. \quad (23)$$

Algorithm (18) can be implemented at the service stations which have communication with neighboring stations. Note that in reality, the driver decides him/herself whether or not he/she wants to charge his/her electric vehicle at a specific station. This stochastic nature of decision-making can be modeled as a perturbation on dynamics (21) and as a result, the states $z_i(k)$ may oscillate around $\xi_0(k)$.

Next, we consider the case where the estimated value $\xi_i((k-1)T + \delta T)$ satisfies

$$\xi_i((k-1)T + \delta T) \geq \sum_{j=1}^N c_j\mu_j. \quad (24)$$

In other words, the estimated current total charging needs of the EVs exceeds the total capacity of the charging stations. We propose the following distributed scheduling algorithm to adjust $p_i(k)$.

$$\begin{aligned} p_1^*(k) &= \frac{c_1\mu_1}{\alpha_1(k)} \xi_0(k) \\ p_i^*(k) &= \frac{c_i\mu_i}{\alpha_i(k)} \{ \xi_0(k) - \eta [z_i(k) - z_{i-1}(k)] \}, \end{aligned} \quad (25)$$

where $i = 2, \dots, N$, $\eta \in (0, 1)$, and

$$\xi_0(k) = \frac{\xi_i((k-1)T + \delta T)}{\sum_{j=1}^N c_j\mu_j}. \quad (26)$$

The following theorem shows that consensus (6) is also guaranteed.

Theorem 2: Under distributed scheduling algorithm (25) and if condition (24) is satisfied, all the charging stations are utilized, i.e., consensus (6) is achieved as $k \rightarrow \infty$ and the queues at service stations will grow.

Proof: Based on condition (24) and function $\hat{g}_i(x_i)$ in (12), the dynamics (20) becomes

$$z_i(k+1) = z_i(k) + \frac{f_i(k)}{c_i\mu_i} - 1. \quad (27)$$

Substituting (25) into (27) yields the closed-loop dynamics:

$$\begin{aligned} z_1(k+1) &= z_1(k) + \xi_0(k) - 1 \\ z_i(k+1) &= (1 - \eta)z_i(k) + \xi_0(k) + \eta z_{i-1}(k) - 1 \end{aligned}$$

Defining and computing $\tilde{z}_{i,l} \triangleq z_i - z_l$ yield

$$\begin{aligned} \tilde{z}_{1,2}(k+1) &= (1 - \eta)\tilde{z}_{1,2}(k) \\ \tilde{z}_{j,j+1}(k+1) &= (1 - \eta)\tilde{z}_{j,j+1}(k) + \eta\tilde{z}_{j-1,j}(k) \end{aligned}$$

where $j = 2, \dots, N-1$. Since $\eta \in (0, 1)$, we have $\tilde{z}_{1,2} \rightarrow 0$ and it follows that $\tilde{z}_{j,j+1} \rightarrow 0$, i.e., consensus (6) is achieved. Furthermore, it follows that

$$f_i(k) \rightarrow c_i\mu_i \xi_0(k). \quad (28)$$

Finally, choosing $\xi_0(k)$ as the one in (26) and computing $\sum_{j=1}^N f_j(k)$ yields

$$\sum_{j=1}^N f_j(k) = \xi_0(k) \sum_{j=1}^N c_j \mu_j = \xi_i((k-1)T + \delta T)$$

which is equal to the condition in (22). In addition, from (24) and (26) we have $\xi_0(k) \geq 1$. Then, from (28) we know that $f_i(k) \geq c_i \mu_i$ and it follows from (27) that the queue at the service stations will grow. ■

Note that from (25) and $z_i(k) \rightarrow \xi_0(k)$, we have

$$p_i^* \rightarrow \frac{c_i \mu_i}{\alpha_i(k)} \xi_0(k). \quad (29)$$

Due to the variation of traffic flow on the highway during the day, the service stations may switch between distributed scheduling algorithms (18) and (25) to minimize the total waiting time of the EVs.

IV. DISTRIBUTED CONTROL OF EVS' CHARGING

After scheduling the number of EVs that need to be charged at each charging stations, the next objective is to design a distributed control $u_{v,i}(k)$ for each EV whose global goal is to meet the optimal percentage of EVs to be charged at each station, i.e. the output of the high-level scheduling while satisfying its own constraint, see figure 4. Given the percentage of EVs that need to be charged at station i , i.e. p_i^* in (23) or (29), the control objective of each EV approaching service station i in (8) can be reformulated as the following optimization problem.

$$\begin{aligned} & \underset{u_{v,i}}{\text{minimize}} && \left[p_i^* - \frac{\sum_v u_{v,i}(k)}{\alpha_i(k)} \right]^2 \\ & \text{subject to} && e_{v,i}^+ \geq d_i r_v^- \\ & && u_{v,i}(k) \in \{0, 1\}. \end{aligned} \quad (30)$$

Note that the first constraint in (30) is introduced to ensure that the EVs will be charged at service station i whenever its battery level is not sufficient to reach the next service station.

In order to solve optimization (30) distributively, the EVs have to negotiate with each other based on their battery level and by means of *vehicle-to-vehicle* communication. We assume that the communication topology between the EVs approaching service station i is given by a strongly connected graph. First, EVs approaching service station i receive the information on $p_i^*(k)$ and $\alpha_i(k)$ broadcasted by service station i by means of *infrastructure-to-vehicle* communication. Using this information, each EV within the set $\mathcal{N}_i(k)$ can independently compute $u_{total}^i(k) = \sum_v u_{v,i}(k) \in \mathbb{Z}$ that minimize $(p_i^* - \frac{u_{total}^i(k)}{\alpha_i(k)})^2$. Without loss of generality, it is assumed that $\dim(\mathcal{N}_i(k)) = \alpha_i(k)$. The EVs then compute their residual battery level $e_{v,i}^-$ and set their control input $u_{v,i}(k) = 1$ if $e_{v,i}^- < d_i r_v^-$. Let the number of EVs that do not satisfy $e_{v,i}^- \geq d_i r_v^-$ be m_i . The rest m_i EVs then sort their $e_{v,i}^-$ values in an ascending order distributively using e.g. [22]. Finally, the EVs with $(u_{total}^i(k) - m_i)$ th smallest $e_{v,i}^-$ set their input $u_{v,i}(k) = 1$. The pseudo-code of the algorithm is presented in algorithm 1.

Algorithm 1 Distributed algorithm to compute (30)

Require: $p_i^*, \alpha_i(k), d_i$, a strongly connected communication topology.

```

1: for  $k = 1, 2, \dots$  do
2:   for  $i = 1, \dots, N$  do
3:     set  $\mathcal{M}_i = \{\}$ 
4:     for  $v = 1, \dots, \alpha_i(k)$  do
5:        $u_{total}^i(k) = \arg \min (p_i^* - \frac{u_{total}^i(k)}{\alpha_i(k)})^2$ 
6:       compute  $e_{v,i}^-$ 
7:       if  $e_{v,i}^- < d_i r_v^-$  then
8:         set  $u_{v,i}(k) = 1$ 
9:          $\mathcal{M}_i \leftarrow v$ 
10:      end if
11:    end for
12:    compute  $m_i = \dim(\mathcal{M}_i)$ 
13:    sort distributively [22] in an ascending order  $e_{v,i}^-$ 
    for  $v \notin \mathcal{M}_i$ 
14:      set  $u_{v,i}(k) = 1$  for  $(u_{total}^i(k) - m_i)$ th smallest  $e_{v,i}^-$ 
15:    end for
16:  end for

```

V. NUMERICAL EXAMPLE

In this section, we illustrate the performance of the proposed distributed scheduling using a numerical example. We consider a highway consisting of 4 service stations. We assume that each service station is equipped with fast chargers, i.e. the service rate for all stations is given by $\mu_i = 2$ (EVs/h) and the number of charging slots at each station is given by $c_1 = 5, c_2 = 4, c_3 = 5, c_4 = 4$. In addition, the distances between the service stations are $d_1 = 2, d_2 = 2, d_3 = 2$. The net average flow at each node is assumed to be constant and equal to $\gamma_1(k) = 12$ (EVs/h), $\gamma_2(k) = 6$ (EVs/h), $\gamma_3(k) = 10$ (EVs/h), $\gamma_4(k) = 5$ (EVs/h) and the average number of charging needed for each EVs is given by $\beta_i = 1 \forall i$.

First, we consider the case when there is no scheduling. In this case, the drivers decide to charge their battery near their maximum driving range. For example, the EVs approaching service station 1 decide to charge their battery at service station 3. As we can see in this case, since $f_3(k) > c_3 \mu_3$ the number of EVs queueing at service station 3 will grow unbounded as time evolves. Next, we apply the distributed scheduling algorithm proposed in section III-C. It is assumed that the distributed estimation of total charging needs, i.e. $\xi_i(kT)$ in (14) have been converged. The results are shown in figures 6 and 7. As illustrated in figure 7, all the service stations are uniformly utilized as time evolves, i.e. $z_i \rightarrow$ constant. This results in that the total waiting time for the EVs at the service stations is minimized. In addition, we can observe that since $\sum_{i=1}^4 \beta_i \gamma_i(k) < \sum_{i=1}^4 c_i \mu_i$, the number of EVs at each service station is bounded at the steady state. As can also be seen from figure 6, in order to balance the utilization of all service stations, most of the EVs originating from node 1 are charged at service stations 1 since its number are the largest among all the nodes.

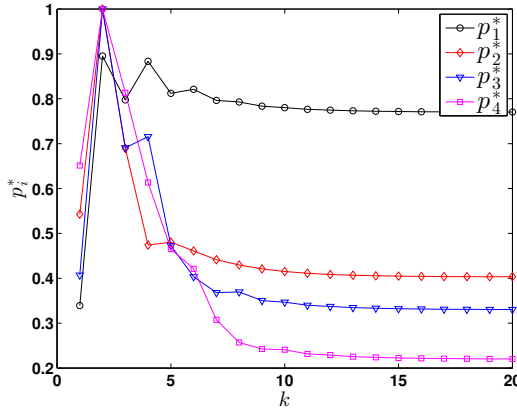


Fig. 6. Proposed distributed scheduling for EVs' charging

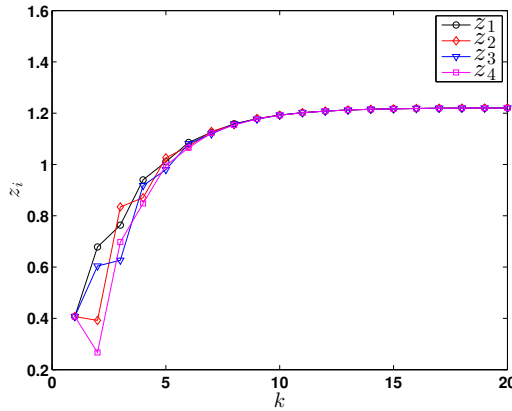


Fig. 7. Utilization of service stations, i.e. $z_i(k) = x_i(k)/c_i\mu_i$ using the proposed distributed scheduling algorithm

VI. CONCLUSION AND FUTURE WORKS

This paper proposes a higher level distributed scheduling algorithm together with a lower level cooperative control policy for individual EVs on a highway in order to optimize the operation of the overall charging network. Specifically, the objective is to make all the charging stations along the highway be well (uniformly) utilized and the total waiting time is minimized. A consensus-based distributed scheduling algorithm is developed which only relies on the information from the neighboring service stations. Furthermore, a negotiation strategy based on the current battery level and among the drivers by means of the vehicle-to-vehicle and vehicle-to-infrastructure communications is presented to meet the published scheduling level. Simulation confirms that the proposed strategy improves the overall system performance. As future works, we aim to investigate the performance of the proposed algorithms in the presence of communication failures between the EVs and with the service stations, influence of human behaviors and also to consider more realistic energy model of the EVs.

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