



# A Simplified Hot Water Distribution System Model

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## A SIMPLIFIED HOT WATER DISTRIBUTION SYSTEM MODEL

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### ABSTRACT

Water heating in the U.S. has been identified as a major component of total energy consumption used in buildings, mostly coming from the residential sector at around 11%. A potential opportunity for energy savings in water heating systems is to improve the design of hot water distribution systems (HWDS). Due to the complex heat losses of HWDS, models are needed to optimize HWDS by reducing heat losses. There are three models currently used to simulate thermal performance of hot water distribution systems (HWDS): HWSim, ORNL-HWDS, and TRNSYS. The first two models are used to study hot water distribution systems only and may not have interactions between a whole building and HWDS. The third model is a whole building approach and uses the “plug-flow” model to calculate the outlet temperature with variable size segments of fluid. The present effort is to develop a simplified HWDS model for a single pipe, which includes thermal mass of both fluid and pipe and can be used in the DOE-2 program as an input function and incorporated in other building simulation programs.

The simplified model is described by a partial differential equation with time and axial distance as independent variables. The model is simplified enough to have an analytical solution and accurate enough to provide a good prediction of HWDS heat losses. The model was validated against measured data during both water heater on and off periods of a water heating system. Following this, an input function used in the DOE-2 program was developed based on the model. Simulation results show that the average domestic hot water heater energy use increases 3%, due to the inclusion of HWDS losses. Heating energy use decreases slightly and cooling energy use increases slightly due to a portion of the heat loss migrating to the conditioned space or adjacent spaces.

### KEYWORDS

Modeling, Hot water distribution system, Energy losses, Simplified HWDS model

### INTRODUCTION

Water heating in the U.S. is a major component of total energy consumption in buildings. In the residential sector water heating is about 11% of the

total (<http://e-center.doe.gov>). The Department of Energy (DOE) lists total primary energy consumption for residential water heating at 2.66 quads. Hot water use in residential buildings accounts for the second largest portion of residential energy consumption in the U.S., second to the energy used for space heating.

It has been estimated that, on average, hot water distribution losses can be in excess of 20% between storage and the end-use point (CEC, 2002). As energy efficiency in buildings improves with technology advances and modern building practices, hot water heating energy can now reach as much as 32% of the energy used on a high performance home (BA 2004). Although the efficiency of water heaters has been mandated by national standards, the efficiency of the distribution system has gone unaddressed. As such, it appears that there is much potential for energy savings in water heating systems by improving and optimizing the design of hot water distribution systems (HWDS).

Many complex factors contribute to heat losses in hot water distribution systems. In addition to the thermal conductivity of the pipe materials used in today's construction (i.e., copper, PEX and CPVC), the environment in which the pipe is routed plays an important role. In a recent study for the California Energy Commission (CEC), Oak Ridge National Laboratory (ORNL) performed detailed simulations of typical HWDS installations and found significant line losses, especially in re-circulating systems (ORNL, 2004).

Due to the complex heat losses of HWDS, models are needed to optimize HWDS. There are three models currently used to simulate thermal performance of hot water distribution systems: HWSim, ORNL-HWDS, and TRNSYS.

The HWSIM model (CEC, 2006), originally developed in 1991 as part of Davis Energy Group's (DEG) original hot water research for the California Energy Commission, has been used since 1992 to develop hot water distribution loss assumptions in California's Residential Standards. The program has significant capabilities but also has shortcomings stemming from the limited scope of the original development effort. In 2004, DEG obtained funding to enhance the program. Key improvements to the model include the ability to simulate distribution system performance under changing environmental

conditions (can adjust inlet cold water temperature and pipe environment temperatures on a monthly basis), improved user interface, and enhanced heat loss algorithms.

ORNL has also developed a numerical model to estimate heat loss or gain from insulated and non-insulated hot water pipes (Wendt et al. 2004), which is arguably the best of several models that have been developed. The required inputs are pipe parameters, insulation properties, and water flow rates. This model calculates energy use, water consumption, and waiting time. The model has been used to evaluate the impacts of alternative HWDS in prototypes of California houses. The model includes thermal mass impacts from water, piping and water flow rates. The model is limited to the study of hot water distribution systems but could be incorporated into a whole building models like DOE-2 (DOE-2 BDL Summary, 1993) and EnergyPlus (<http://www.energyplus.gov>). Since it is a very detailed model, more efforts are needed for this model to be integrated in a whole building simulation program.

TRNSYS (2000) has a pipe or duct model (Type 31, “plug-flow”), using variable size segments. The mass of the new segment is equal to the flow rate times the simulation time step. The outlet temperature is averaged with mass weight by including the fluid thermal mass only. A new model (Type 306) was also developed by DLR (2006) with a lumped capacity model. TRNSYS is a whole building simulation tool and able to simulation interactions between a building and the HDWSs.

The National Association of Home Builders (NAHB) used TRNSYS to estimate energy consumption for hot water systems and to further simulate other system design options (NAHB 2002). The simulation model was calibrated with heat-transfer coefficients determined by experimental results. It was used to evaluate the use of demand water heating equipment in conjunction with various hot water piping configurations.

The first two models are used to study hot water distribution systems only and may not be used for a whole building approach. The third model is able to simulate a whole building with HWDS losses; however, it does not include the important thermal mass impacts of pipes and the outlet fluid temperature is averaged based on weighted mass in different segments. In addition, since variable segments are used to calculate the outlet temperature, it needs more computational time.

The present effort is to develop a simplified HWDS model for a single pipe that includes dynamic impacts of both fluid and pipes. The model should be simplified enough to have an analytical solution and accurate enough to provide a good prediction of HWDS heat losses. The main purpose is to make the

model easily integrated in building simulation programs, such as an input function in DOE-2, to examine impact of HWDS losses on whole building energy use.

## HWDS MODEL DEVELOPMENT

The following simplifying assumptions are used in the model development:

- Water temperature is constant at a given cross section
- When a copper pipe is used, conductive resistance through the copper pipe wall is assumed to be negligible in the radial direction
- Water and copper pipe have the same temperature at a given cross section.
- Water and copper pipe temperature is a function of axial distance from the hot water source and the length of time the outlet (faucet or shower) is activated.
- Water and copper pipe temperature is a function of time only for a period following the time an outlet (faucet or shower) is deactivated.
- Insulation has no thermal capacity.
- Convective heat transfer coefficient on the air side at the external surface is independent of temperature and time.

The simplified governing equation to calculate heat losses in HWDS is divided into on and off periods. The HWDS on period is the fixture activated time, including waiting time for usable hot water to arrive at a fixture. The HWDS off period is the deactivated (standby) time. The difference is that there is water flow during the on period, while no water flow during the off period. Since it is possible to have both on and off periods during a simulation time step restricted by a whole building simulation program, it needs to include energy losses in both periods.

### HWDS On

The governing equation during the water heater on period is given below:

$$(\dot{m}C_p)_w \frac{\partial T}{\partial x} + \left[ (\rho_w C_{p,w} A_w) + (\rho_p C_{p,p} A_p) \right] \frac{\partial T}{\partial \tau} + UPT = UPT_{\infty} \quad (1)$$

where U is the overall heat transfer coefficient [W/m<sup>2</sup>.K], including pipe insulation without thermal mass impact

$$U = \frac{1}{\frac{1}{h_o} + \sum \frac{t_j}{k_j}} \quad (2)$$

Boundary condition: T(0,t) = T<sub>inlet</sub> [°C]

Initial condition: T(x,0) = T<sub>a</sub> [°C]

Let

$$a_m = (\dot{m}C_p)_w$$

$$a_\rho = (\rho_w C_{p,w} A_w) + (\rho_p C_{p,p} A_p)$$

The above governing equation can be solved numerically and analytically. The difference from both solutions will be addressed in a later section.

#### Numerical solution

Since the governing equation is a partial differential equation with respect to axial distance and time, the equation may be solved numerically using the following finite difference method:

$$T_{i,c} = \frac{a_m * T_{i-1,c} / \Delta x + a_\rho * T_{i,p} / \Delta \tau + UPT_\infty}{a_m / \Delta x + a_\rho / \Delta \tau + UP} \quad (3)$$

#### Analytical solution

The Laplace transform was used to solve the first order partial differential equation by assuming the boundary conditions remain unchanged. This is equivalent to employing unit step function. The temperature distribution in a pipe is expressed below:

$$T(x, t) = T_\infty + \left[ (T_{inlet} - T_\infty) * \exp\left(-\frac{UPx}{a_m}\right) - (T_a - T_\infty) * \exp\left(-\frac{UPt}{a_\rho}\right) \right] * u\left(t - \frac{a_\rho}{a_m}x\right) + (T_a - T_\infty) * \exp\left(-\frac{UPt}{a_\rho}\right) \quad (4)$$

where  $u(t)$  is a unit step function and may be written as

$$u = \begin{cases} = 0 & \text{when } t < \frac{a_\rho}{a_m}x \\ = 1 & \text{when } t > \frac{a_\rho}{a_m}x \end{cases}$$

#### Heat losses

$$Q_{loss,on} = \int_0^{t_{on}} \int_0^L [UP(T(x, \tau) - T_\infty)dx] d\tau = \int_0^{t_{init}} \int_0^L [UP(T(x, \tau) - T_\infty)dx] d\tau + \int_{t_{init}}^{t_{on}} \int_0^L [UP(T(x, \tau) - T_\infty)dx] d\tau \quad (5)$$

where

$$\int_0^{t_{init}} \int_0^L [UP(T(x, \tau) - T_\infty)dx] d\tau = \int_0^{t_{init}} \frac{a_m \tau}{a_\rho} \int_0^L [UP(T(x, \tau) - T_\infty)dx] d\tau + \int_0^{t_{init}} \int_0^L [UP(T(x, \tau) - T_\infty)dx] d\tau =$$

$$L * a_\rho * (T_a - T_\infty) * \left[ 1 - e^{-\frac{UP}{a_\rho} t_{init}} \right] + a_m * (T_{inlet} - T_\infty) * \left[ t_{init} + \frac{a_\rho}{UP} \left( e^{-\frac{UP}{a_\rho} t_{init}} - 1 \right) \right] + (T_a - T_\infty) * \frac{a_m a_\rho}{UP} \left[ e^{-\frac{UP}{a_\rho} t_{init}} * \left( \frac{UP}{a_\rho} t_{init} + 1 \right) - 1 \right] \quad (6)$$

$$\int_{t_{init}}^{t_{on}} \int_0^L [UP(T(x, \tau) - T_\infty)dx] d\tau = \quad (7)$$

$$UP(t_{on} - t_{init}) \int_0^L (T_{inlet} - T_\infty) * \exp\left(-\frac{UPx}{a_m}\right) dx = a_m (t_{on} - t_{init}) * (T_{inlet} - T_\infty) * \left( 1 - \exp\left(-\frac{UPL}{a_m}\right) \right) \quad (8)$$

$$t_{init} = \frac{a_\rho}{a_m} x$$

#### HWDS Off

The governing equation during water heater off period is given below with mass flow rate is set to zero:

$$a_\rho \frac{\partial T}{\partial \tau} + UPT = UPT_\infty \quad (9)$$

Initial condition:  $T(0) = T_{init}$

#### Analytical solution

$$T(t) = T_\infty + (T_{init} - T_\infty) * \exp\left(-\frac{UPt}{a_\rho}\right) \quad (10)$$

#### Heat losses

$$Q_{heat,off} = \left[ (\rho_w C_{p,w} A_w) + (\rho_p C_{p,p} A_p) \right] * (T_{init} - T_{\infty}) * \left[ 1 - \exp\left(-\frac{UPt}{a\rho}\right) \right] * L \quad (11)$$

The governing equation during the off period is the same as one used in Hiller's work (2005).

### HWDS MODEL VALIDATION

The measurement was performed in a re-circulated hot water distribution system in a Florida residential home with the following parameters used in model validation (Vieira et al. 2006):

- Copper pipe diameter = 0.019 m (0.75 in)
- Pipe length = 23.5 m (77 ft)
- Inlet temperature = 55 °C (131 °F)
- Initial temperature = 43.3 °C (110 °F)
- Water flow rate = 0.00013 m<sup>3</sup>/s (2 gpm)
- Ambient temperature = 32.2 °C (90 °F)
- On time: 180 sec
- Off time: 5 minutes

The data were collected at every second during the on period, and at every minute during the off period. The ambient temperature is the temperature surrounding the pipes.

Figure 1 shows measured and predicted temperatures at the shower outlet before and during shower activation. The predicted temperatures were obtained from both numerical and exact solutions. Although the numerical approach is only an approximation, it is very close to the measured data. The accuracy is dependent on the magnitude of  $\Delta x$  and  $\Delta \tau$ . Note that the exact solution indicates a temperature jump at time = 63 ( $a_p/a_m x$ ) seconds instead of a slow temperature change. This occurs because, based on the simplifying assumptions, a unit function is used when the time is greater than  $t_2 = a_p/a_m x$ . In reality, the outlet temperature should rise when the hot water reaches the outlet at  $t > t_1 (= xA/Q)$ , until the outlet temperature reaches the steady state condition at  $t > t_3 (= 2t_2 - t_1)$ . In this case, the numerical approach provides the better solution for temperature prediction. From a total energy loss perspective by integrating temperature with respect to time, both solutions provide similar results. The difference of the integrated areas between 40 and 63 seconds is equal to the difference of the integrated areas between 63 ( $t_2$ ) and 78 ( $t_3$ ) seconds. The energy loss during the shower on time (180 seconds) is 6246.633 J from the numerical solution, and 6246.603 J from the exact solution.

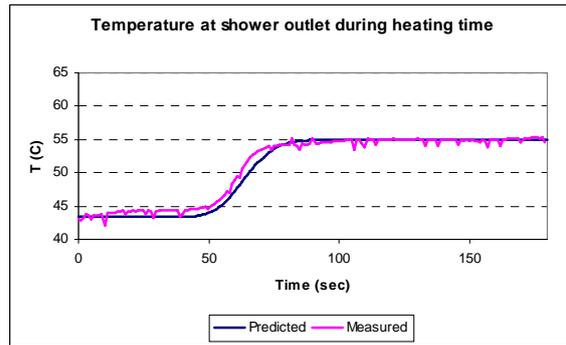


Figure 1. Temperature comparison between measurement and prediction at shower outlet during heating time

Figure 2 plots the temperature comparison between the same measurement and prediction at the shower outlet after the shower is turned off. Since it is easy to obtain an exact solution, no numerical approach is needed. As shown in the figure, the data match quite well.

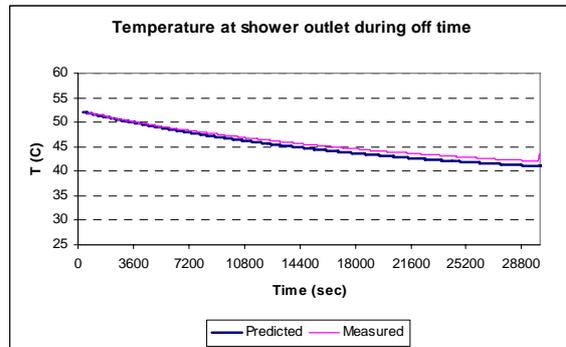


Figure 2. Temperature comparison between measurement and prediction at shower outlet during heating off period

### HWDS MODEL APPLICATION

Even though simplifying assumptions are used for the governing equations, the model validation shows that the model can predict the temperature distribution and energy losses very well. The next step is to integrate the model into a whole building simulation program, so that energy losses from HWDS becomes a part of zone sensible loads and additional energy use for a hot water heater in a whole building. DOE-2 is selected as a whole building simulation program.

The input function is named as DHWLOADS, and is called in the Zone section before the zone calculation is performed in the system computation. The required input values are:

- Tank size [gal]
- Tank water set temperature [°F]
- Pipe diameter [in]
- Pipe length [ft]
- Water flow rate [gal/min]
- Thermal resistance of pipe insulation [R]

- Copper pipe thickness [in]
- Water use schedule

It should be pointed out that the units in the above inputs are IP units, consistent with the other input units.

The calculation procedure uses exact solutions for both heating on and off periods in the following steps:

1. Check if the water heater is on or off based on a given water heater operation schedule
2. If the water heater is off, calculate pipe heat losses based on Eq. (11)
3. If the water heater is on, calculate pipe heat losses based on Eq. (5) during the on time fraction, then calculate pipe heat losses based on Eq. (11) during the off time fraction
4. Save pipe temperature for the next time step use
5. Add the pipe energy loss into DHWKW, which is a variable for hot water heater energy use in DOE-2
6. Add the pipe energy loss into the zone sensible load

Note: When hot water heater is on during the whole hour, the pipe heat losses are calculated for one hour. When the hot water heater is on for a fraction of the hour, it is assumed that the heater is on from the beginning of the hour, and off during the rest of the hour.

Due to HWDS losses, the interactions between HWDS and HVAC systems involve adding hot water heater energy use and zone sensible loads. It is assumed that the HWDS losses are immediately released to the zone at the same time step. In reality, pipes are located in cavities of interior walls. The thermal mass of interior walls causes delay of loss release. However, the sum of total HWDS losses between immediate and delay releases are similar during long periods of time, thus there is little impact in annual simulations. Since the HWDS losses are added before zone the load calculation is performed, space heating and cooling energy uses are changed accordingly.

A typical energy efficient residential building with a 186 m<sup>2</sup> (2000 ft<sup>2</sup>) conditioned space was simulated with and without HWDS energy losses using seven locations, representing U.S. climate zones 1 through 6 (ASHRAE Standard 90.2). TMY2 weather data in the seven locations were used in the simulations.

Table 1 lists annual simulation results of hot water heater and total energy use in the seven locations, extracted from Report BEPS in units of kWh. The first column lists location. Columns 2 and 3 present domestic hot water heater and total energy use without HWDS losses. Columns 4 and 5 present domestic hot water heater and total residence energy use with HWDS losses. The last two columns show percent changes of domestic hot water heater and

total energy use with HWDS losses compared to those without HWDS losses. The domestic hot water heater energy use increase with HWDS energy losses averaged in seven locations is 3%. The total energy use changes vary with locations. Since HWDS losses add more heat in the conditional space, the losses cause space cooling increase and space heating decrease, so that the total energy use change is based on the sum of space heating and cooling changes. For a cooling dominated climate like Miami, the total energy use increase 1.6%. For a heating dominated climate like Boston, the total energy use decreases 1.1%. Figure 3 provides the percent changes of domestic hot water heater and total energy use in seven locations.

Table 1: Building energy performance summary with and without HDWS losses

Location	No HWDS		HWDS		% Change	
	DHW	Total	DHW	Total	DHW	Total
Miami	2462	8968	2520	9114	2.38	1.63
Houston	2491	8851	2579	8939	3.53	0.99
Atlanta	2520	9613	2579	9613	2.33	0.00
Baltimore	2520	11928	2608	11840	3.49	-0.74
Boston	2550	13511	2608	13364	2.30	-1.08
Minneapolis	2550	18141	2638	18024	3.45	-0.65
San Francisco	2550	8851	2638	8821	3.45	-0.33

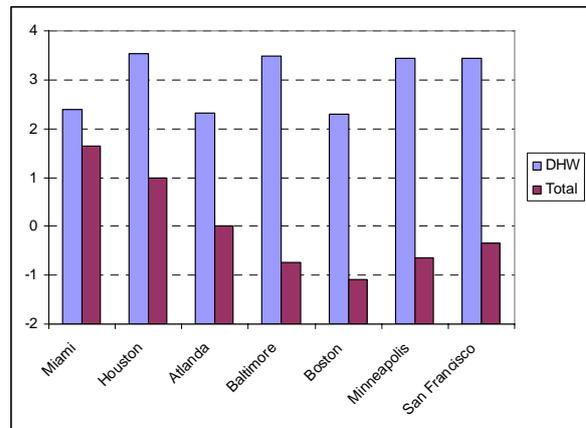


Figure 3 Percent changes of domestic hot water heater and total energy use with and without HDWS losses in seven locations

## CONCLUSIONS AND RECOMMENDATIONS

The following conclusion can be drawn:

- A simplified model to calculate HWDS energy losses, including thermal capacity impact of pipes and water, was developed. The model was validated against limited measured data and was integrated into a whole building simulation program to calculate the impact of HWDS energy losses on whole building energy use.
- Although the code is written as an input function of DOE-2, the input function can be used as a general function to calculate HWDS losses, as long as required inputs are available.
- Due to the limitation of DOE-2, the input function developed is only able to simulate straight piping of the same size. A complex HWDS may not be simulated.
- The present model is restricted with boundary conditions exposed to surrounding air.
- Although the model is applied to a single pipe, the model can be easily enhanced to simulate a multiple pipe system.
- The model is applied to the HWDS system exposed to surrounding air with uniform temperatures.

#### Recommendations

- Since the governing equations are simple enough to be easily integrated into a network model to calculate heat losses in a realistic HWDS, it is possible to simulate a complex HWDS with real configuration.
- Further model validation is needed including a whole building, when more measured data are available.
- As long as correct boundary conditions are used, the model is expected to be used with HWDS buried under ground.
- When pipes are located in a wall cavity, localized boundary conditions may be needed to predict energy losses more accurately, since the cavity temperature may not be the same as the zone temperature.

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#### NOMENCLATURE

$A_w$	= Water flow area [ $m^2$ ]
$A_p$	= Pipe cross section area [ $m^2$ ]
$C_{p,w}$	= Water specific heat [ $J/kg.K$ ]
$C_{p,p}$	= Pipe specific heat [ $J/kg.K$ ]
$h_o$	= Heat transfer coefficient at the exterior pipe surface [ $W/m^2.K$ ]

$k_i$	= Thermal conductivity at i-th radial layer of a pipe [ $W/m.K$ ]
$\bullet$	
$m$	= Water flow rate [ $kg/s$ ]
$P$	= Pipe perimeter [ $m$ ]
$Q$	= Volumetric flow rate [ $m^3/s$ ]
$t_i$	= Thickness at i-th radial layer of a pipe (normal to the axial direction) [ $m$ ]
$T$	= Pipe and water temperature as a function of axial distance and time [ $^{\circ}C$ ]
$T_{i,c}$	= Water temperature at ith node and current time step
$T_{i,p}$	= Water temperature at ith node and previous time step
$T_{\infty}$	= Surrounding air temperature where a pipe is located [ $^{\circ}C$ ]
$x$	= Pipe axial distance [ $m$ ]
$\Delta x$	= The distance between ith and (i+1)th node ( $L/200$ is used in numerical solution)
$\Delta \tau$	= The time difference between previous time step and current time step
$\rho_w$	= Water density [ $kg/m^3$ ]
$\rho_p$	= Pipe density [ $kg/m^3$ ]
$\tau$	= Time [ $s$ ]

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