APPENDIX E
Derivation of Part Load Efficiency Function

Derivation of Part Load Factor

The goal is to find PLF = F(CLF, N_{max}, τ)

First, define:

$$X = \frac{t_{ON}}{t_{cycle}} \qquad \text{(E-1)}$$

where: t_{ON} = time AC is ON
t_{cycle} = time for a complete ON & OFF cycle

The thermostat cycling equation (from Appendix A) is defined as:

$$N = 4N_{max}X(1-X) \qquad \text{(E-2)}$$

where: N = $1/t_{cycle}$, the number of ON/OFF cycles per hour
N_{max} = Maximum cycle rate

Rearranging equation (E-2) results in:

$$t_{ON} = \frac{1}{4N_{max}(1-X)} \qquad \text{(E-3)}$$

The response of the an AC system is approximately first-order and can be represented as:

$$Q = Q_{ss}(1 - e^{-t/\tau}) \qquad \text{(E-4)}$$

where: τ = Time constant of AC system (time)
Q, Q_{ss} = AC cooling capacity (energy/time)

Integrating Q over t_{ON}:

$$q = \int_0^{t_{ON}} Q\, dt$$
$$q = Q_{ss}(t_{ON} - \tau(1 - e^{-t_{ON}/\tau})) \qquad \text{(E-5)}$$

Then defining:

$$Q_{avg} = \frac{q}{t_{cycle}} \qquad \text{(E-6)}$$

$$EER_{avg} = \frac{q}{E_{ss} t_{ON}} \quad \text{(E-7)}$$

$$EER_{ss} = \frac{Q_{ss}}{E_{ss}} \quad \text{(E-8)}$$

$$CLF = \frac{Q_{avg}}{Q_{ss}} = \frac{\text{Load}}{\text{AC Capacity}} \quad \text{(E-9)}$$

$$PLF = \frac{EER_{avg}}{EER_{ss}} = \frac{\text{Part Load EER}}{\text{Steady State EER}} \quad \text{(E-10)}$$

Combining (E-5) with (E-6) and (E-9) results in:

$$CLF = \frac{t_{ON}}{t_{cycle}} - \frac{\tau}{t_{cycle}}(1 - e^{-t_{ON}/\tau}) \quad \text{(E-11)}$$

Combining (E-5) with (E-7) and (E-10) results in:

$$PLF = 1 - \frac{\tau}{t_{ON}}(1 - e^{-t_{ON}/\tau}) \quad \text{(E-12)}$$

Comparing (E-11) and (E-12) results in:

$$X = \frac{t_{ON}}{t_{cycle}} = \frac{CLF}{PLF} \quad \text{(E-13)}$$

Finially, by substituting (E-3) and (E-13) into (E-12) results in:

$$PLF_{i+1} = 1 - 4\tau N_{max}(1 - CLF/PLF_i)[1 - e^{\frac{-1}{4\tau N_{max}(1 - CLF/PLF_i)}}] \quad \text{(E-14)}$$

Since PLF occurs on both sides of equation (E-14), iterations are necessary to find PLF.

The part load curve from the SEER test procedure is:

$$PLF = 1 - C_D(1-CLF) \qquad \text{(E-15)}$$

Where C_D is equal to 0.25 by default.

By setting CLF=0, and comparing equations (E-14) and (E-15):

$$C_D \approx 4\tau N_{max}\left(1 - e^{\frac{-1}{4\tau N_{max}}}\right) \qquad \text{(E-16)}$$

From the default values used in the cyclic tests in the SEER procedure, τ and N_{max} can be shown to be:

τ = 76 seconds (0.0212 hours)
N_{max} = 3.125 cycles/hour

This results in:

C_D = 0.258

This is very close to the default value of 0.25 used in the SEER test procedure.